Exponential Growth

An ecological system is vulnerable to destruction from the introduction of plants or animals outside that system. For example, when a non-native plant is introduced into a new area, it usually does not have its growth checked by the environment. With unlimited food and no predators, the non-native plant grows exponentially, quickly covering, killing, and replacing the endemic, native, plants. Such a plant is the water hyacinth which has been called the worst aquatic plant in the world! It is native to South America, but is now a part of the lakes, rivers and streams in most of the southern United States.

The water hyacinth is said to have been introduced into the United States in Louisiana at the World's Industrial and Cotton Centennial Exposition of 1884-1885. A Floridian went home with water hyacinth plants and later released them into the St. Johns River. From this beginning, water hyacinths now cause problems such as forming impenetrable mats of floating vegetation. These block boat traffic, prevent swimming and fishing, and prevent sunlight and oxygen from getting into the water. Perhaps the worst effect of the water hyacinth is a reduction of biological diversity in waters which it inhabits.

1. Water hyacinths double in 6 to 18 days. For the purposes of this investigation, consider that the number of water hyacinths will double in 12 days. Also assume that there is unlimited food and that there are no predators. Suppose 4 water hyacinths were put into the St. John's River. In question 5 you will make a table to figure out how many water hyacinths were produced from these 4 at a given time. Determine in your group the information in this problem that will be important in determining the number.

2. After 4 doubling periods, how many days will have elapsed? **Be sure to justify each answer in this investigation.**

3. After 96 days, there will have been how many doubling periods?

4. In 60 days, how many times will the water hyacinth population double? In 120 days?

5. Complete the following table to show the number of water hyacinths from these four at the given time intervals.

Days	Doubling Periods	Process Column	Exponential Version	Total Water Hyacinths
0	0 - Starting point	4	$4 \cdot 2^0$	4
12	1	$4 \cdot 2$	$4 \cdot 2^1$	8
24	2			
	3			
	4			
	5			
	6			
	10			

6. Develop a pattern which you can use to fill in the "days" and "total water hyacinths" columns below. The total water hyacinths will be a "formula" which includes the starting number of water hyacinths, the fact that the number is doubling, and the number of times that the number has doubled.

Days	Doubling Periods	Process Column	Exponential Version	Total Water Hyacinths
n				

7. About how many days would it take for there to be more than a billion from the original 4? How many years?

8. Using your formula, how many water hyacinths would there be after 3 years? (Ignore leap years.) Be sure to justify your answer in the space below.

9. Suppose growing conditions for water hyacinths were ideal and they doubled every 6 days. Rewrite your formula so that you start with the same four water hyacinths and double every 6 days.

10. Using the new formula, how many water hyacinths will there be after 3 years? (Still ignore leap years and justify your answer.)

11. Do you think that the formula will accurately predict the total water hyacinths indefinitely, or are there environmental conditions which might limit it? If you are the environmental scientist in charge of reducing the population, what will you try to keep the water hyacinths from completely clogging the waterways?

12. Non-native plants can destroy an ecosystem. Non-endemic animals can do the same. In 1859, 22 rabbits were imported to Australia from Europe. The rabbits found a land with plenty of food and no natural predators. Soon there were so many rabbits that they were destroying the island. In Australia, the rabbit population was doubling every 6 months. Determine in your group the information above that will be important in determining the number of rabbits at any given time.

13. Explain how to calculate the number of years elapsed after four doubling periods.

14. How many doubling periods will have elapsed after 18 months?

15. How many times has the rabbit population doubled after 4 years? After 7.5 years?

16. Complete the following table to show the number of rabbits from these 22 at the given time intervals.

Months	Growth Periods	Process Column	Exponential Version	Total Rabbits
0	0 - Starting point	22		
6	1			
	2			
	3			
	4			
	6			
	8			
	10			

17. In the table below, write an exponential function which can be used to determine the total number of rabbits after n growth periods in months.

Months	Growth Periods	Process Column	Exponential Version	Total Rabbits
n				

18. In 1887 there were so many rabbits that the government of Australia offered a reward for a way to control the population. How many growth periods had passed by 1887?

19. Approximately how many rabbits were there by 1887? Describe how you calculated your answer.

20. What population controls would you have offered?

21. Let a = the starting amount, b = the growth factor for a given amount of time, and n = number of doubling periods. Write an exponential function, in terms of n, which can be used for any exponential growth situation.